Matroid theory
Exercise Sheet 6
Date: 17 March 2026

Exercise 6.1. Show that if b is a polymatroid function and $M(b)$ is the matroid associated with it, then $b(X) = b(\overline{X})$, where $\overline{X}$ denotes the closure of X in $M(b)$.

Exercise 6.2. Show that every matching matroid is a transversal matroid.

Exercise 6.3. Prove that every graphic matroid is representable over any field.

Mock midterm

Problem 1. *Let C_1 and C_2 be circutis of a matroid $M = (S, \mathcal{F})$ such that $C_1 \cup C_2 = S$ and $C_1 - C_2 = \{x\}$. Prove that for every circuit $C \neq C_1$, we have $C \supseteq C_2 - C_1$.*

Problem 2. *Prove that in a matroid $M = (S, \mathcal{F})$ the intersection of any two flats is a flat.*

Problem 3. *Prove that in a matroid $M = (S, \mathcal{F})$, if $x \in C(B, y) - y$, then $C(B - x + y, x) = C(B, y)$.*

Problem 4. *Let $M = (S, \mathcal{F})$ be a matroid. Give an algorithm for deciding if there exists a basis that has maximum weight with respect to all of the weight functions $c_1, \ldots, c_k$.*

Problem 5. *Prove that $M/X = M \setminus X$ if and only if $M = M|X \oplus M|(S - X)$.*

Problem 6. *Give an example showing that the dual of a paving matroid is not necessarily paving.*

Problem 7. *Prove that in a matroid $M = (S, r)$, every flat X is the intersection of $r(S) - r(X)$ hyperplanes.*

Homework (submission deadline: March 24)

Exercise 6.4. Let $M = (S, r)$ be a transversal matroid defined by the graph $G = (S, T; E)$. Assume that M^* has no loops and T has no isolated elements. Prove that $|T| = r(S)$.

Challenging problem (submission deadline: April 7)

Exercise 6.5. A matroid M is *strongly base orderable* if for any two bases A, B there exists a bijection $\varphi : A \to B$ such that

$$A - X + \varphi(X) \text{ is a basis for every } X \subseteq A.$$

Let M be a strongly base orderable matroid of rank n whose ground set is partitioned into n disjoint bases $B_1, \ldots, B_n$. Then there exist n pairwise disjoint transversal bases, where a basis is *transversal* if it intersects B_i for $i = 1, \ldots, n$.

Research opportunity

Conjecture 6.6. Let M be a matroid of rank n whose ground set is partitioned into n disjoint bases $B_1, \ldots, B_n$. Then there exist n pairwise disjoint transversal bases.