**Deep learning and continuous optimization**

Submission deadline:
19 March, 14:00

**Exercise 1** (2pts)**.** Let $g(x) = x^2 - a$. Show that Newton's method, when computing the root of $g$, leads to the recurrence $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$.

**Exercise 2** (2pts)**.** Consider the problem of minimizing the function $f(x) = x \log x$ over $x \in \mathbb{R}_{>0}$. Perform the full convergence analysis of Newton's method - consider all starting points $x_0 \in \mathbb{R}_{>0}$ and determine where the method converges for each of them.

**Exercise 3** (2pts)**.** Formulate the following problem as a convex optimization problem: Find the box

$$\mathcal{B} = \{x \in \mathbb{R}^n \mid l \leq x \leq u\}$$

of maximum volume, enclosed in a polyhedron $\mathcal{P} = \{x \in \mathbb{R}^n \mid Ax \leq b\}$. The variables are $l, u \in \mathbb{R}^n$. Your formulation should not involve an exponential number of constraints.

**Exercise 4** (2pts)**.** Prove the following statements.

1. **Reflection:** If $f$ is submodular, then $g(X) \coloneqq f(S \setminus X)$ is submodular.
2. **Contraction:** If $X \subseteq S$ and $f$ is submodular, then $g(Y) \coloneqq f(X \cup Y) - f(X)$ is submodular.

**Exercise 5** (2pts)**.** Consider the two-element set $S = \{a, b\}$, and let $f \colon 2^S \to \mathbb{R}$ be defined as $f(\emptyset) = 0$, $f(\{a\}) = 2$, $f(\{b\}) = 1$, and $f(\{a, b\}) = 2$. Determine the Lovász-extension of $f$ for any vector $c = (c_1, c_2) \in \mathbb{R}^2$.