Deep learning and continuous optimization

Submission deadline:

12 March, 14:00

Exercise 1 (2pts)**.** Let $f : \mathbb{R}^n \to \mathbb{R}$ a function and define $\tilde{f} : \mathbb{R}^n \to \mathbb{R}$ as $\tilde{f}(x) = f(Ax + b)$ where $A \in \mathbb{R}^{n \times n}$ is an invertible matrix and $b \in \mathbb{R}^n$. Verify that if x_0 moves to x_1 by applying one step of Newton's method with respect to $\tilde{f}$, then $y_0 = Ax_0 + b$ moves to $y_1 = Ax_1 + b$ by applying one step of Newton's method with respect to f.

Exercise 2 (2pts)**.** Let $f(x_1, x_2) = x_1^2 + Kx_2^2$ be a quadratic function where $K > 0$. Define $\|(u_1, u_2)\|_\circ := \sqrt{u_1^2 + Ku_2^2}$. Determine the optimum point of $\max\limits_{\|u\|_\circ \leq 1} -\langle \nabla f(x), u \rangle$.

Exercise 3 (2pts)**.** Let $f(x) = x^T Ax + b^T x$ (where A is positive definite). Define $\|u\|_A := \sqrt{u^T Au}$. Determine the optimum point of $\max\limits_{\|u\|_A \leq 1} -\langle \nabla f(x), u \rangle$.

Exercise 4 (2pts)**.** Let $f(x) : \mathbb{R}^n \to \mathbb{R}$ a strictly convex function (i.e., the Hessian $\nabla^2(x)$ is positive definite at every point $x \in \mathbb{R}^n$). Determine the optimum point of $\max\limits_{\|u\|_x \leq 1} -\langle \nabla f(x), u \rangle$.

Exercise 5 (2pts)**.** Prove that $\frac{1}{2}\|n(x_0)\|_{x_0}^2$ is the difference between the value of f at x_0 and the minimum value of the second order quadratic approximation of f at x_0.