Matroid theory
Matroid or not?
Date: 10 February 2026

Decide whether each of the following structures always defines a matroid. Unless stated otherwise, S denotes a finite, nonempty set, $k \leq |S|$ is a positive integer, and $G = (V, E)$ is a finite, simple, connected graph. The structures are given alternately via independent sets, bases, circuits, or rank functions, denoted by $\mathcal{F}$, $\mathcal{B}$, $\mathcal{C}$, and r, respectively.

(M1) Ground set: S; $r(X) = |X|$ for all $X \subseteq S$.

(M2) Ground set: S; $r(X) = 0$ for all $X \subseteq S$.

(M3) Ground set: S; $F \in \mathcal{F}$ if $s \in F$ for some fixed $s \in S$.

(M4) Ground set: S; $B \in \mathcal{B}$ if $|B| = k$.

(M5) Ground set: $\{s_1, s_2, s_3, s_4, s_1', s_2', s_3', s_4'\}$; $B \in \mathcal{B}$ if $|B| = 4$ and B is not one of the following sets: $\{s_1, s_2, s_1', s_2'\}, \{s_1, s_3, s_1', s_3'\}, \{s_1, s_4, s_1', s_4'\}, \{s_2, s_3, s_2', s_3'\}, \{s_2, s_4, s_2', s_4'\}$.

(M6) Ground set: S, let $S = P_1 \cup P_2 \cup \ldots \cup P_t$ be a partition of S; $F \in \mathcal{F}$ if $|F \cap P_i| \leq 1$ for $1 \leq i \leq t$.

(M7) Ground set: S, let $S = P_1 \cup P_2 \cup \ldots \cup P_t$ be a partition of S; $C \in \mathcal{C}$ if $C = P_i$ for some $1 \leq i \leq t$.

(M8) Ground set: S, let $\sim$ be an equivalence relation on S with at least one equivalence class of size at least 2; $B \in \mathcal{B}$ if $B = \{s_1, s_2\}$ and $s_1 \sim s_2$.

(M9) Ground set: S, let $H_1, H_2, \ldots, H_t \subseteq S$ be k-element sets with $|H_i \cap H_j| \leq k - 2$ for $1 \leq i < j \leq t$; $B \in \mathcal{B}$ if $|B| = k$ and $B \neq H_i$ for all $1 \leq i \leq t$.

(M10) Ground set: E; $r(X) = |V| - c(X)$ for all $X \subseteq E$ where $c(X)$ is the number of connected components of the graph $G_X = (V, X)$.

(M11) Ground set: E; $C \in \mathcal{C}$ if C is an inclusionwise minimal cut of G.

(M12) Ground set: E; $F \in \mathcal{F}$ if F contains at most one cycle.

(M13) Ground set: E; $F \in \mathcal{F}$ if F contains no even cycle, and at most one odd cycle.

(M14) Ground set: E; $F \in \mathcal{F}$ if for every $X \subseteq V$ induces at most $2|X| - 3$ edges in the graph $G_F = (V, F)$.

(M15) Ground set: E; $F \in \mathcal{F}$ if F is the union of two forests.

(M16) Ground set: E; $B \in \mathcal{B}$ if $|B| = k$ and it contains no triangle.

(M17) Ground set: E; $F \in \mathcal{F}$ if F is a matching.

(M18) Ground set: V; $F \in \mathcal{F}$ if F can be covered with a matching.

(M19) Ground set: V; $F \in \mathcal{F}$ if F is an independent set (i.e., no two points in F are connected with an edge).

(M20) Ground set: V_1, where $G = (V_1, V_2, E)$ is a bipartite graph; $F \in \mathcal{F}$ if F can be covered with a matching.

(M21) Ground set: the set of edges incident with a fixed vertex v in G, let u be another fixed vertex; $F \in \mathcal{F}$ if there are $|F|$ edge-disjoint $v - u$ paths, each containing an edge from F.

(M22) Ground set: A, where $G = (V, A)$ is a directed graph; $F \in \mathcal{F}$ if F forms an arborescence.

(M23) Ground set: E, let $w : E \to \mathbb{R}_+$ be a weight function; $B \in \mathcal{B}$ if B is a maximum weight spanning tree.

(M24) Ground set: S, let $w : S \to \mathbb{R}_+$ be a weight function; $F \in \mathcal{F}$ if $\sum_{s \in F} w(s) \leq 1$.

(M25) Ground set: Finite set of points in the plane; $C \in \mathcal{C}$ if $|C| \geq 3$ and there exists a circle containing exactly the points of C.

(M26) Ground set: Finite set of points in the space, not all lying on a plane; $B \in \mathcal{B}$ if $|B| = 4$ and the points in B are the vertices of a (non-degenerate) tetrahedron.

(M27) Ground set: Columns of a real matrix; $F \in \mathcal{F}$ if the corresponding columns are linearly independent.

(M28) Ground set: Finite set of events; $F \in \mathcal{F}$ if the events in F are mutually independent.

(M29) Ground set: Elements of a finite group Γ; $B \in \mathcal{B}$ if B is an inclusionwise minimal generator of Γ.

(M30) Ground set: $\{1, 2, \ldots, n\}$ for some $n \geq k$, let $1 \leq a_1 < a_2 < \ldots < a_k \leq n$; $B \in \mathcal{B}$ if $|B| = k$ and the i-th smallest element of B is at least a_i for all $1 \leq i \leq k$.