Deep learning and continuous optimization
Submission deadline:
19 February, 14:00

Exercise 1 (2pts)**.** Consider the problem $x_2 \leq 4, x_1 + x_2 \leq 6, 2x_1 + x_2 \leq 10, x_1, x_2 \geq 0$. Represent these constraints on the plane. Find a point that maximizes $x_1 + 2x_2$.

Exercise 2 (1pt)**.** Let $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $c_1, \ldots, c_k \in \mathbb{R}^n$. Formulate the following problem as an LP: $Ax = b$, $x \geq 0$, $\min f(x)$, where $f(x) := \max\{c_1 x, \ldots, c_k x\}$.

Exercise 3 (2pts)**.** A company is manufacturing k different products using m resources. The amounts of available resources are given, together with the requirement of each of them for the different products. The selling price of the products are also known.

(a) Write up an IP model that aims at maximizing the total profit.

(b) Adjust the model if starting the production of product i requires a cost of s_i.

Exercise 4 (2pts)**.** Consider the integer programming problem

$$\begin{aligned} \text{minimize} \quad & x_{n+1} \\ \text{subject to} \quad & 2x_1 + 2x_2 + \cdots + 2x_n + x_{n+1} = n \\ & x_i \in \{0, 1\} \end{aligned}$$

Show that any branch and bound algorithm that uses LP relaxations to compute lower bounds, and branches by setting a fractional variable to either zero or one, will require the enumeration of an exponential number of subproblems when n is odd.

Exercise 5 (2pts)**.** The pagination problem faced by a document processing program like LaTeX can be abstracted as follows. The text consists of a sequence $1, \ldots, n$ of n items (words, formulas, etc.). A page that starts with item i and ends with item j is assigned an attractiveness factor c_{ij}. Assuming that the factors c_{ij} are available, we wish to maximize the total attractiveness of the paginated text. Develop an algorithm for this problem. (Hint: try to use a recursive approach.)

Exercise 6 (1pt)**.** Is it true that a set $K \subseteq \mathbb{R}^n$ is convex if and only if for any $x, y \in K$ we have $(x + y)/2 \in K$?