Matroid theory
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Exercise 1. Develop a min-max theorem for the maximum weight of a common independent set of i elements.

Exercise 2. Let M_1, M_2 be matroids over the same ground set S, and let $S_1, S_2 \subseteq S$. Give an algorithm that decides if there exist bases $B_1 \in \mathcal{B}_1$ and $B_2 \in \mathcal{B}_2$ such that $B_1 - B_2 \subseteq S_1$, $B_2 - B_1 \subseteq S_2$.

Exercise 3. Given a finite set of points in a two-dimensional plane, find the maximum number of disjoint triples each of which defines a non-degenerate triangle.

Exercise 4. Let M be a connected matroid over a set S with $|S| \geq 2$. Prove that at least one of M/e and $M \backslash e$ is connected.

Exercise 5. Let M be a matroid. Prove that M is uniform if and only if every circuit of M meets every cocircuit of M.

Exercise 6. Let $M = (S, \mathcal{B})$ be a matroid and let $C \subseteq S$ be a circuit of M that is a hyperplane as well. Prove that $\mathcal{B}' = \{B \cup C \mid B \in \mathcal{B}\}$ forms the family of bases of a matroid.

Exercise 7. Prove that each simple planar graph can be covered by three forests.

Exercise 8. Prove that if M is a cographic matroid without coloops, then $\beta(M) \leq 3$.

Exercise 9. Prove that if G is a 4-edge-connected graph then G is the union of two bipartite subgraphs.