Road surveillance optimization — an asymmetric vehicle routing problem with visiting frequencies

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Abstract: This paper discusses a real life optimization problem related to route planning of public road surveillance vehicles. The public roads must be checked regularly with a legally determined visiting frequency depending on the road category and traffic intensity. The surveillance is made by multiple vehicles starting from and arriving at their own garages, which take place at different locations. The traveling and surveying speed are different, and these predefined speed values also vary with the type of the road. Finally, the daily maximum working time is also limited. Then, the goal it to minimize the total travel time or distance of the surveillance vehicles while taking all the parameters and side constraints above. The problem may be considered as a Period Vehicle Routing Problem with a highly asymmetric length function. The paper discusses several variants of the problem depending on the allowed visiting patterns of each road segment and on whether or not the roads are preassigned to the surveillance vehicles. Heuristic and integer linear programming based schemes are presented to solve these large scale optimization problems. The proposed algorithms are tested on real life problem instances in a framework of a pilot project initiated by the Hungarian Public Road Nonprofit Private Limited Company.

Keywords: periodic vehicle routing, large scale optimization

1 Introduction

This paper presents a short summary of a mathematical optimization project aiming at optimizing the daily route plans of surveillance vehicles. The project was initiated by the Hungarian Public Road Nonprofit Private Limited Company.

Public roads* must regularly be surveyed in order to check the status of the pavement, traffic signs, cleanness etc. The checking includes both human observations and measurements and it is done by a special surveillance vehicle. The public road company has several such a vehicles at different base locations (called *centers*). The regularity of the surveillance is mandated by the law and depends on the road category and average traffic (the usual values being 1, 2, 7, 14 or 28 days). The speed of the vehicles depends on whether it is actually surveying or just traveling, but these speed values are well predictable for each route segments. Naturally the total daily working time of each vehicle is limited. The goal is to find a cyclic daily routing plan for the vehicle for a given time interval, e.g. for 28 day. The (optimal) amount of useful travel is determined by the input, but there always will be redundant travel between two roads to be surveyed. The goal is simply to minimize the total travel of the vehicles.

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*There are also special objects to be surveyed, such as bridges or crossings, but mathematically they can be converted to short piece of roads

Thus the problem is well described by the road networks with the parameters above, and it can be classified as a Vehicle Routing Problem[1] where the locations must be visited regularly instead of just once. Similar problems are discussed in the literature under the name *Period Vehicle Routing Problem*, see e.g.[4]. A significant difference is however we want to visit (and go through) roads instead of just a simple location. This makes our cost function highly asymmetric, e.g. the “distance” from road segments $u \rightsquigarrow v$ to $v \rightsquigarrow w$ should be considered 0, but not from $v \rightsquigarrow w$ to $u \rightsquigarrow v$. For some practical approaches to the related Asymmetric Traveling Salesmen problem, the reader is referred to [2] and [3].

Remaining of the paper is organized as follows. Section 2 provides the exact specification of the problem, which is then transformed to a MIP formulation in Section 3. This formalization already allows are to solve small to medium problem instances with open source are commercial solvers. For larger problem instances a cutting plane based and a purely heuristic approach is presented in Section 4 and 5, respectively.

2 Problem Statement

In mathematical form, the problem can be defined as follows. We are given a directed graph $G = (V, E)$ representing the road network, a set $\mathcal{C} := \{C_1, C_2, \ldots, C_k\} \subset V$ of center nodes (i.e. the base of each surveillance vehicles), the length $R \in \mathbb{N}$ of the planning interval, the $W \in \mathbb{R}$ maximum daily working time of the vehicles and the following edge attributes.

$freq : E \longrightarrow \mathbb{N}$ determines the maximum (or exact) number of days between two consecutive checks of the road segment. $freq(e) = 0$ means that e does not have to be surveyed (but can be used for traveling).

$time_{trv} : E \longrightarrow \mathbb{R}$ **and** $time_{srv} : E \longrightarrow \mathbb{R}$ denote the time needed to cross the road segments when just traveling or when the road is actually surveyed, respectively.

$length : E \longrightarrow \mathbb{R}$ is the physical length of the road segments.

$bidir : E \longrightarrow \{0, 1\}$ indicates whether or not the road segment is bidirectional.

$centers : E \longrightarrow 2^{\mathcal{C}}$ is the set of vehicles (i.e. centers) which are allowed to survey the corresponding road segment.

With all these notations, our goal is to find a $p_{c,d}$ (closed) paths and a set $S_{c,d} \subset p_{c,d}$ of surveyed roads for each center $c \in \mathcal{C}$ and day $d \in [0, dots, T-1]$ so that

- $p_{c,d}$ starts from and ends at c,
- $S_{c,d} \subset \{e : c \in centers(e)\}$, i.e. each road is surveyed by one of its allowed vehicles,
- $\sum_{e \in S_{c,d}} time_{srv}(e) + \sum_{e \in p_{c,d} \setminus S_{c,d}} time_{srv}(e) \leq W$, i.e. the total travel time of $p_{c,d}$ is at most W,
- $e \in \bigcup\{S_{c,k} : c \in \mathcal{C}, k \in \{d, (d+1)_{modR}, \ldots, (d+freq(e)-1)_{modR}\}$ holds for each $e \in E$, $freq(e) > 0$, and $d \in [0, \ldots, R-1]$.

While keeping all the constraints above, we seek to minimize the total length of all paths $p_{c,d}$, i.e

$$\sum_{c \in \mathcal{C}, d \in [0, \ldots, R-1]} length(p_{c,d}). \tag{1}$$

3 Mixed Integer Programming Formulation

This section gives a Mixed Integer Programming formulation of the problem specified in Section 2, which is capable to provide optimal solution to small to moderate problem instances.

$$\min \sum_{cde} len(e) s_{cde} + \sum_{cde} len(e) t_{cde} \tag{2}$$

where

$$s_{cde} \in \{0,1\} \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], e \in E_c \tag{3}$$

$$t_{cde} \in \mathbb{N} \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], e \in E_c \tag{4}$$

$$f_{cde} \geq 0 \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], e \in E_c \tag{5}$$

$$\sum_{e \in \rho_{E_c}(v)} s_{cde} + \sum_{e \in \rho_E(v)} t_{cde} = \sum_{e \in \delta_{E_c}(v)} s_{cde} + \sum_{e \in \delta_E(v)} t_{cde} \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], v \in V \tag{6}$$

$$f_{cde} \leq M(s_{cde} + t_{cde}) \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], e \in E_c \tag{7}$$

$$\sum_{e \in \rho_E(v)} f_{cde} - \sum_{e \in \delta_E(v)} f_{cde} = \sum_{e \in \rho_{E_c}(v)} s_{cde} \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], v \in V - c \tag{8}$$

$$\sum_{k \in \{d,\dots,(d+freq(e)-1)_{modR}\}} s_{cke} \geq 1 \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], e \in E_c \tag{9}$$

$$\sum_{e \in E_c} time_{srv}(e) s_{cde} + \sum_{e \in E} time_{trv}(e) t_{cde} \leq W \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1] \tag{10}$$

In this formulation, variables s_{cde} and t_{cde} indicate the survey and non-survey edges of the path belonging to center c and day d. Equations (6)– (8) together guarantee that the edges belonging to a path form a connected Eulerian subgraph. The survey frequency of the edges and the maximum daily travel times are forced by Equation (9) and (10), respectively.

4 Cutting Plane Approach

The most problematic part of the formulation described in Section 3 are the connectivity constraints (7) and (8), mainly due to used M constant, which inevitably introduces a large integrality to the problem therefore makes the Branch&Bound scheme inefficient. The usual way to overcome this difficulty is to replace the flow based connectivity constraints to a cut based formulation consisting of exponentially many implicit constraints, which are added the problem only when they are violated during the optimization. (see. e.g. [5]). This idea would lead to replacing Equations (7) and (8) with

$$\sum_{e \in \delta_{E_c}(U)} s_{cde} + \sum_{e \in \delta_E(U)} t_{cde} \geq \frac{1}{M} \sum_{(uv) \in E_C; u,v \in U} s_{cd(uv)} \qquad \forall c \in \mathcal{C}, d \in [0,\dots,R-1], U \subset V - c \tag{11}$$

Unfortunately, it still uses the M constant, and indeed does not improve a lot on the performance of the Branch&Bound scheme. Instead we propose a much more effective cutting plane approach, at the cost that it can exclude some feasible solution.

Namely, whenever the MIP solver uses a feasible integer solution, we — in a callback function executed be the solver — check the connectivity of each route. If it is found to be unconnected, i.e. there exists a cut $U \subset V - c$ such that S_{cd} intersects the edges spanned by U but $(S_{cd} \cup S_{cd}) \cap (U, V \setminus U) = \emptyset$, then we add the following constraint to the system.

$$\sum_{e \in \delta_{E_c}(U)} s_{cde} + \sum_{e \in \delta_E(U)} t_{cde} \geq 1 \tag{12}$$

5 Heuristic Approaches

This section briefly sketches two simple heuristics capable of providing solutions comparable to the cutting plane method described in Section 4 in a fraction of time.

In further restrict the set of feasible solutions, namely we require that if the visiting frequency of a road is k that is visited exactly in every k^{th} day. Thus a visiting pattern of a road can simply be characterized by its offset $offs(e) \in [0, \ldots, freq(e) - 1]$

5.1 Insertion Heuristic

We start from an empty path p_{cd} for each center c and day d, then add each road segment one-by-one. For inserting road e, we first calculate the cheapest insertion of e into each route, choose the cheapest route for each day, then find the cheapest offset to which e can be inserted.

The working time constraint is enforced by increasingly penalizing the paths that are approaching to the working time limit.

The obtained solution can further be improved by trying to remove each road e and reinsert it optimally as described above. Then this process can be repeated until no more improvement can be found.

5.2 Simulated Annealing

This algorithm start from an arbitrary feasible solution and in each iteration it applies a road reinsertion similarly to the one described above. However, worsening changes are also allowed here.

Namely, at each iteration we choose a random road e and day offset o. Then remove e form the current solution then reinsert it optimally to offset o. Finally we decide randomly whether we accept the change or revert to the previous one. The acceptance probability is given by the formula

$$P(accept) := max \left\{ 1, e^{\frac{cost_{prev} - cost_{new}}{T}} \right\}, \tag{13}$$

where $cost_{prev}$ and $cost_{new}$ are the cost of the previous and the new solutions, while T is the so-called *temperature* parameter, that decreasing exponentially during the algorithm (which is called *annealing*), making the acceptance criterion of worsening changes stricter.

The interested reader is referred to [6] for the motivation behind the algorithm and for more details on the temperature control and stop condition.

In general, one can observe that the Simulated Annealing algorithm runs longer than the Insertion Heuristic, but can provide better solution. In addition, one can balance between fast execution and closer-to-optimal solutions by the careful choice of starting temperature, the speed of annealing and the stop condition.

6 Summary

This paper proposed different approaches to solve a special Vehicle Routing Problem related to route planning of public road surveillance vehicles. Firstly, a direct MIP formalization is shown to calculated exact optimal solution to smaller problem instances. Then, it is augmented with a cutting plane approach in order to make it suitable for larger inputs. Finally, two heuristics were proposed that are able to provide good quality solutions rapidly even for the largest realistic problems.

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